課名：統計學一 104 學年度第一學期 期末考題 (總共 100 points)

姓名：__________________ 學號：____________________

一、選擇題 (共 40 points)

1. An unbiased estimator of a population parameter is defined as:
   a. an estimator whose variance is equal to one.
   b. an estimator whose expected value is equal to zero.
   c. an estimator whose variance goes to zero as the sample size goes to infinity.
   d. None of these choices.

2. The letter α in the formula for constructing a confidence interval estimate of the population mean is:
   a. the level of confidence.
   b. the probability that a particular confidence interval will contain the population mean.
   c. the area in the lower tail of the sampling distribution of the sample mean.
   d. None of these choices.

3. In developing an interval estimate for a population mean, a sample of 50 observations was used. The interval estimate was 19.76 ± 1.32. Had the sample size been 200 instead of 50, the interval estimate would have been:
   a. $19.76 \pm .33$
   b. $19.76 \pm .66$
   c. 19.76 ± 5.28
   d. None of these choices.

4. In developing an interval estimate for a population mean, the population standard deviation σ was assumed to be 10. The interval estimate was 50.92 ± 2.14. Had σ equaled 20, the interval estimate would be
   a. 60.92 ± 2.14
   b. 50.92 ± 12.14
   c. 101.84 ± 4.28
   d. 50.92 ± 4.28

5. Suppose a 95% confidence interval for μ turns out to be (1,000, 2,100). What does it mean to be 95% confidence?
   a. In repeated sampling, the population parameter would fall in the resulting interval 95% of the time.
   b. 95% of the observations in the entire population fall in the given interval.
   c. 95% of the observations in the sample fall in the given interval.
   d. None of these choices.

6. The sample size needed to estimate a population mean to within 10 units was found to be 68. If the population standard deviation was 50, then the confidence level used was:
   a. 99%
   b. 95%
   c. 90%
   d. None of these choices.

7. We cannot commit a Type I error when the:
   a. null hypothesis is true.
   b. level of significance is 0.10.
   c. null hypothesis is false.
   d. test is a two-tail test.

8. Suppose we wish to test $H_0: \mu = 45$ vs. $H_1: \mu > 45$. What will result if we conclude that the mean is greater than 45 when the actual mean is 50?
   a. We have made a Type I error.
   b. We have made a Type II error.
   c. We have made both a Type I error and a Type II error.
   d. We have mad the correct decision.

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9. The owner of a local Jazz Club has recently surveyed a random sample of $n = 200$ customers of the club. She would now like to determine whether or not the mean age of her customers is over 30. If so, she plans to alter the entertainment to appeal to an older crowd. If not, no entertainment changes will be made. The appropriate hypotheses to test are:
   a. $H_0: \mu = 30$ vs. $H_1: \mu < 30$.
   b. $H_0: \mu = 30$ vs. $H_1: \mu > 30$.
   c. $H_0: \bar{X} = 30$ vs. $H_1: \bar{X} < 30$.
   d. $H_0: \bar{X} = 30$ vs. $H_1: \bar{X} > 30$.

10. In order to determine the *p*-value, which of the following is not needed?
   a. The level of significance.
   b. Whether the test is one-tail or two-tail.
   c. The value of the test statistic.
   d. All of these choices are true.

11. If a hypothesis is not rejected at the 0.10 level of significance, it:
   a. must be rejected at the 0.05 level.
   b. may be rejected at the 0.05 level.
   c. will not be rejected at the 0.05 level.
   d. must be rejected at the 0.025 level.

12. Using a confidence interval when conducting a two-tail test for μ, we do not reject H_0 if the hypothesized value for μ:
   a. is to the left of the lower confidence limit (LCL).
   b. is to the right of the upper confidence limit (UCL).
   c. falls between the LCL and UCL.
   d. falls in the rejection region.

13. Suppose that in a certain hypothesis test the null hypothesis is rejected at the .10 level; it is also rejected at the .05 level; however, it cannot be rejected at the .01 level. The most accurate statement than can be made about the *p*-value for this test is that:
   a. *p*-value $= 0.01$.
   b. *p*-value $= 0.10$.
   c. $0.01 < p$-value < 0.05.
   d. $0.05 < p$-value < 0.10.

14. We have created a 95% confidence interval for μ with the results (10, 25). What conclusion will we make if we test $H_0: \mu = 26$ vs. $H_1: \mu \neq 26$ at $\alpha = 0.025$?
   a. Reject H_0 in favor of H_1
   b. Accept H_0 in favor of H_1
   c. Fail to reject H_0 in favor of H_1
   d. We cannot tell from the information given.

15. Based on sample data, the 90% confidence interval limits for the population mean are LCL = 170.86 and UCL = 195.42. If the 10% level of significance were used in testing the hypotheses $H_0: \mu = 201$ vs. $H_1: \mu \neq 201$, the null hypothesis:
   a. would be rejected.
   b. would be accepted.
   c. would fail to be rejected.
   d. None of the above.

16. A random sample of 25 observations is selected from a normally distributed population. The sample variance is 10. In the 95% confidence interval for the population variance, the upper limit is:
   a. 19.353
   b. 17.331
   c. 17.110
   d. 6.097

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17. In a hypothesis test for the population variance, the hypotheses are $H_0: \sigma^2 = 30$ vs. $H_1: \sigma^2 < 30$. If the sample size is 20 and the test is being carried out at the 5% level of significance, the null hypothesis is rejected if :
    a. $\mathcal{X}^2 < 30.144$.
    b. $\mathcal{X}^2 > 10.851$.
    c. $\mathcal{X}^2 < 10.117$.
    d. $\mathcal{X}^2 > 31.410$.
18. In selecting the sample size to estimate the population proportion *p*, if we have no knowledge of even the approximate values of the sample proportion $\hat{p}$, we:
    a. take another sample and estimate $\hat{p}$.
    b. take two more samples and find the average of their $\hat{p}$.
    c. let $\hat{p} = 0.50$.
    d. let $\hat{p} = 0.95$.
19. After calculating the sample size needed to estimate a population proportion to within 0.04, your statistics professor told you the maximum allowable error must be reduced to just .01. If the original calculation led to a sample size of 800, the sample size will now have to be:
    a. 800
    b. 3,200
    c. 6,400
    d. 12,800
20. A survey claims that 9 out of 10 doctors recommend aspirin for their patient with headaches. To test this claim against the alternative that the actual proportion of doctors who recommend aspirin is less than 0.90, a random sample of 100 doctors' results in 83 who indicate that they recommend aspirin. The value of the test statistic in this problem is approximately equal to:
    a. -1.67
    b. -2.33
    c. -1.86
    d. -0.14

二、計算題：(共 60 points)

1. (16 points) A company claims that 10% of the users of a certain allergy drug experience drowsiness. In clinical studies of this allergy drug, 81 of the 900 subjects experienced drowsiness.
   (1) (2 points) We want to test their claim and find out whether the actual percentage is not 10%. State the appropriate null and alternative hypotheses.

   (2) (4 points) Is there enough evidence at the 5% significance level to infer that the company is correct?

(3) (4 points) Compute the *p*-value of the test.

(4) (4 points) Construct a 95% confidence interval estimate of the population proportion of the users of this allergy drug who experience drowsiness?

(5) (2 points) Explain how to use this confidence interval to test the hypotheses.

2. (12 points) Engineers who are in charge of the production of springs used to make car seats are concerned about the variability in the length of the springs. The springs are designed to be 500 mm long. When the springs are too long, they will loosen and fail out. When they are too short, they will not fit into the frames. The springs that are too long and too short must be reworked at considerable additional cost. The engineers have calculated that a standard deviation of 2 mm will result in an acceptable number of springs that must be reworked. A random sample of 100 springs was measured, and the sample variance is equal to 6.52.

(1) (6 points) Can we infer at the 5% significance level that the number of springs requiring reworking is **<u>unacceptably large</u>**?

(2) (6 points) Suppose the engineers reduced the data so that springs that were the correct length were recorded as 1, springs that were too long were recorded as 2, and springs that were too short were recorded as 3. The engineers found that there are 86 springs recorded as 1, 6 springs recorded as 2, and 8 springs recorded as 3. Can we infer at the 10% significance level that less than 90% of the springs are the correct length?

3. (16 points) A researcher wants to study the average lifetime of a certain brand of light bulbs (in hours). In testing the hypotheses, $H_0: \mu = 950$ hours vs. $H_1: \mu \neq 950$ hours, a random sample of 25 light bulbs is drawn from a normal population whose standard deviation is 200 hours.

(1) (5 points) Calculate β, the probability of a Type II error when $\mu = 1000$ and $\alpha = 0.10$.

(2) (4 points) Calculate the power of the test when $\mu = 1000$ and $\alpha = 0.10$.

(3) (4 points) Recalculate β if n is increased from 25 to 40.

(4) (3 points) Recalculate β if α is lowered from 0.10 to 0.05.

4. (9 points) A statistics professor would like to estimate a population mean to within 40 units with 99% confidence given that the population standard deviation is 200.

(1) (3 points) What sample size should be used?

(2) (2 points) What sample size should be used if the standard deviation is changed to 50?

(3) (2 points) What sample size should be used if using a 95% confidence level?

(4) (2 points) What sample size should be used if we wish to estimate the population mean to within 10 units?

5. (7 points) An accountant was performing an audit for a tractor dealership. An auditor wants to examine the monetary error made by the purchasing order department in the month of July. He decided to randomly sample 100 of the 925 purchase orders for the month of July, and found the amount of error in each one. The statistics for this sample were: $\bar{x} = \$6.0$ and $s = \$17.012$.

(1) (4 points) Estimate with 95% confidence the average amount of error per purchase order for the entire month of July. (Hint: Should the finite population correction factor needs to be used?)

(2) (3 points) Estimate with 95% confidence the total amount of monetary error for the month of July.

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Solution

一

DDBDA　CCDBA　CCC<u>D(C)</u>A　ACCDB

二

1. (1) $H_0: P = 0.1$ $H_1: P \neq 0.1$
   (2) Don't reject H_0
   (3) *p*-value: 0.3174
   (4) LCL: 0.071　UCL: 0.109
   (5) Fail to reject H_0 at $\alpha = 0.050$
2. (1) $\mathcal{X}^2 = 161.25$　*p*-value$= 0.0001$
   There is enough evidence to conclude that at the 10% significance level that less than 90% of the springs are the correct length.
   (2) $\mathrm{z} = -1.33$, *p*-value$= -0.0912$.
   There is enough evidence to conclude that at the 10% significance level that less than 90% of the springs are the correct length.
3. (1) $\beta = 0.6535$
   (2) power: 0.3465
   (3) $\beta = 0.5239$
   (4) $\beta = 0.7611$
4. (1) $n =$ 163 / 165 / 167
   (2) $n = 11$
   (3) $n = 97$
   (4) $n =$2653 / 2663
5. (1) $\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = 6 \pm 1.984 \times \frac{17.012}{\sqrt{100}} \sqrt{\frac{925-100}{925-1}}$
   (2) $N\left[\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}\right] = 925\left[6 \pm 1.984 \times \frac{17.012}{\sqrt{100}} \sqrt{\frac{925-100}{925-1}}\right]$